Math at Work

When I was a kid, I always liked seeing how things work. I read books to see how things worked and I liked to tinker with things like bicycles, roller skates, or whatever. So it was only natural that I would become a mechanical engineer.

I design methods of controlling turbine speed, so a turbine can be used to generate electric power. The turbine is pushed by steam. A valve is opened to let the steam in, and the speed of the turbine is dictated by how much steam is allowed into the system. Once the turbine is spinning it generates electricity, which runs the pumps, which determine the position of the valves. In this way, the turbine actually controls its own speed.

We use oil pressure to make all of this happen. Oil cannot be compressed, so when we push oil inside a tube, the oil opens the valve, which determines how much steam is pushing the turbine. We need the pumps to push the oil, which controls the opening or closing of the valve. The turbine needs to be able to get to speed in a given amount of time, as well as shut down fast enough in case of emergency, and the opening and closing of the valve controls this speed.

While computers do much of the math these days, we use calculus for stress and motion analysis, as well as vibration analysis, fluid flow, and heat analysis. One of the equations we use is:

John Jay

$$\gamma_1 + \frac{v_1^2}{2} + gh_1 = \gamma_2 + \frac{v_2^2}{2} + gh_2$$

This is an *energy balance equation*. The γ's represent density of the oil, the v's represent flow velocity of the oil, the g's represent the force of gravity, and the h's represent the height of the oil above the point where it exerts pressure. The subscript 1's indicate these quantities at one point in the system, and the subscript 2's indicate these quantities at another point in the system.

What this equation means is that the total energy at one point in the system must equal the total energy at another point in the system. Thus, we know how much energy is exerted against the valve controls, wherever they are located in the system.

CHAPTER 1 Key Ideas

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CHAPTER 1 Review Exercises

The collection of exercises marked in red could be used as a chapter test.

In Exercises 1–10, match the graph with the corresponding function (a)–(j) from the list below. Use your knowledge of function behavior, *not* your grapher.

(a) $f(x) = x^2 - 1$ **(b)** $f(x) = x^2 + 1$
(c) $f(x) = (x - 2)^2$ **(d)** $f(x) = (x + 2)^2$
(e) $f(x) = \frac{x-1}{2}$ **(f)** $f(x) = |x - 2|$
(g) $f(x) = |x + 2|$ **(h)** $f(x) = -\sin x$
(i) $f(x) = e^x - 1$ **(j)** $f(x) = 1 + \cos x$

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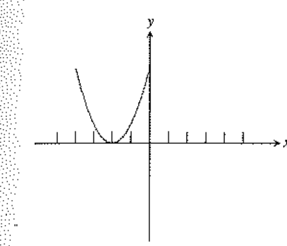
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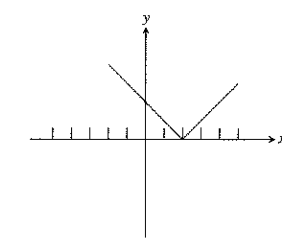
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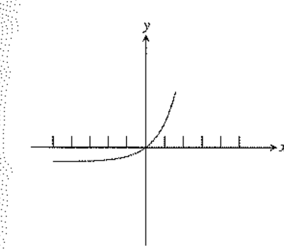
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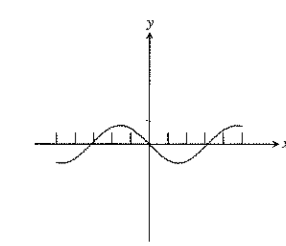
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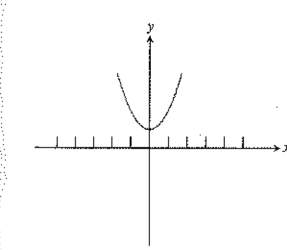
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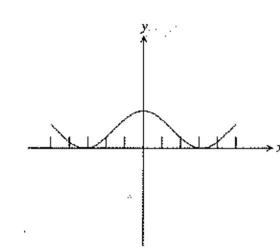
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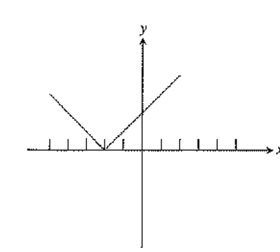
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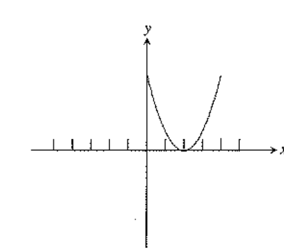
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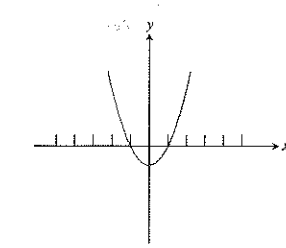
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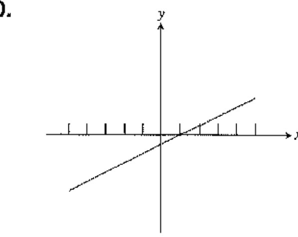


In Exercises 11–18, find **(a)** the domain and **(b)** the range of the function.

11. $g(x) = x^3$ **12.** $f(x) = 35x - 602$
13. $g(x) = x^2 + 2x + 1$ **14.** $h(x) = (x - 2)^2 + 5$
15. $g(x) = 3|x| + 8$ **16.** $k(x) = \sqrt{4 - x^2} - 2$
17. $f(x) = \frac{x}{x^2 - 2x}$ **18.** $k(x) = \frac{1}{\sqrt{9 - x^2}}$

In Exercises 19 and 20, graph the function, and state whether the function is continuous at $x = 0$. If it is discontinuous, state whether the discontinuity is removable or nonremovable.

19. $f(x) = \frac{x^2 - 3}{x + 2}$ **20.** $k(x) = \begin{cases} 2x + 3 & \text{if } x > 0 \\ 3 - x^2 & \text{if } x \le 0 \end{cases}$

In Exercises 21–24, find all **(a)** vertical asymptotes and **(b)** horizontal asymptotes of the graph of the function. Be sure to state your answers as equations of lines.

21. $y = \frac{5}{x^2 - 5x}$ **22.** $y = \frac{3x}{x - 4}$
23. $y = \frac{7x}{\sqrt{x^2 + 10}}$ **24.** $y = \frac{|x|}{x + 1}$

In Exercises 25–28, graph the function and state the intervals on which the function is *increasing*.

25. $y = \frac{x^3}{6}$ **26.** $y = 2 + |x - 1|$
27. $y = \frac{x}{1 - x^2}$ **28.** $y = \frac{x^2 - 1}{x^2 - 4}$

In Exercises 29–32, graph the function and tell whether the function is bounded above, bounded below, or bounded.

29. $f(x) = x + \sin x$ **30.** $g(x) = \frac{6x}{x^2 + 1}$
31. $h(x) = 5 - e^x$ **32.** $k(x) = 1000 + \frac{x}{1000}$

In Exercises 33–36, use a grapher to find all **(a)** relative maximum values and **(b)** relative minimum values of the function. Also state the value of x at which each relative extremum occurs.

33. $y = (x + 1)^2 - 7$ **34.** $y = x^3 - 3x$
35. $y = \frac{x^2 + 4}{x^2 - 4}$ **36.** $y = \frac{4x}{x^2 + 4}$

In Exercises 37–40, graph the function and state whether the function is odd, even, or neither.

37. $y = 3x^2 - 4|x|$

38. $y = \sin x - x^3$

39. $y = \dfrac{x}{e^x}$

40. $y = x\cos(x)$

In Exercises 41–44, find a formula for $f^{-1}(x)$.

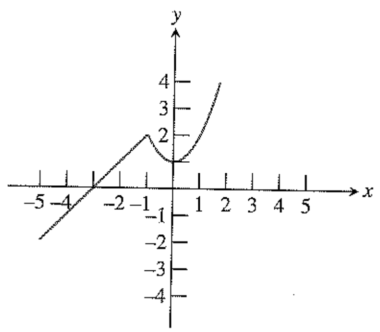
41. $f(x) = 2x + 3$

42. $f(x) = \sqrt[3]{x - 8}$

43. $f(x) = \dfrac{2}{x}$

44. $f(x) = \dfrac{6}{x + 4}$

Exercises 45–52 refer to the function $y = f(x)$ whose graph is given below.



45. Sketch the graph of $y = f(x) - 1$.

46. Sketch the graph of $y = f(x - 1)$.

47. Sketch the graph of $y = f(-x)$.

48. Sketch the graph of $y = -f(x)$.

49. Sketch a graph of the inverse relation.

50. Does the inverse relation define y as a function of x?

51. Sketch a graph of $y = f(|x|)$.

52. Define f algebraically as a piecewise function. [*Hint*: the pieces are translations of two of our "basic" functions.]

In Exercises 53–58, let $f(x) = \sqrt{x}$ and let $g(x) = x^2 - 4$.

53. Find an expression for $(f \circ g)(x)$ and give its domain.

54. Find an expression for $(g \circ f)(x)$ and give its domain.

55. Find an expression for $(fg)(x)$ and give its domain.

56. Find an expression for $\left(\dfrac{f}{g}\right)(x)$ and give its domain.

57. Describe the end behavior of the graph of $y = f(x)$.

58. Describe the end behavior of the graph of $y = f(g(x))$.

In Exercises 59–64, write the specified quantity as a function of the specified variable. Remember that drawing a picture will help.

59. Square Inscribed in a Circle A square of side s is inscribed in a circle. Write the area of the circle as a function of s.

60. Circle Inscribed in a Square A circle is inscribed in a square of side s. Write the area of the circle as a function of s.

61. Volume of a Cylindrical Tank A cylindrical tank with diameter 20 feet is partially filled with oil to a depth of h feet. Write the volume of oil in the tank as a function of h.

62. Draining a Cylindrical Tank A cylindrical tank with diameter 20 feet is filled with oil to a depth of 40 feet. The oil begins draining at a constant rate of 2 cubic feet per second. Write the volume of the oil remaining in the tank t seconds later as a function of t.

63. Draining a Cylindrical Tank A cylindrical tank with diameter 20 feet is filled with oil to a depth of 40 feet. The oil begins draining at a constant rate of 2 cubic feet per second. Write the depth of the oil remaining in the tank t seconds later as a function of t.

64. Draining a Cylindrical Tank A cylindrical tank with diameter 20 feet is filled with oil to a depth of 40 feet. The oil begins draining so that the depth of oil in the tank decreases at a constant rate of 2 feet per hour. Write the volume of oil remaining in the tank t hours later as a function of t.

65. U.S. Crude Oil Imports The imports of crude oil to the U.S. from Canada in the years 1995–2004 (in thousands of barrels per day) are given in Table 1.15.

Table 1.15 Crude Oil Imports from Canada

Year	Barrels/day × 1000
1995	1,040
1996	1,075
1997	1,198
1998	1,266
1999	1,178
2000	1,348
2001	1,356
2002	1,445
2003	1,549
2004	1,606

Source: Energy Information Administration, Petroleum Supply Monthly, as reported in The World Almanac and Book of Facts 2005.

(a) Sketch a scatter plot of import numbers in the right-hand column (y) as a function of years since 1990 (x).

(b) Find the equation of the regression line and superimpose it on the scatter plot.

(c) Based on the regression line, approximately how many thousands of barrels of oil would the U.S. import from Canada in 2010?

66. The winning times in the women's 100-meter freestyle event at the Summer Olympic Games since 1952 are shown in Table 1.16:

Table 1.16 Women's 100-Meter Freestyle

Year	Time	Year	Time
1952	66.8	1980	54.79
1956	62.0	1984	55.92
1960	61.2	1988	54.93
1964	59.5	1992	54.64
1968	60.0	1996	54.50
1972	58.59	2000	53.83
1976	55.65	2004	53.84

Source: The World Almanac and Book of Facts 2005.

(a) Sketch a scatter plot of the times (y) as a function of the years (x) beyond 1900. (The values of x will run from 52 to 104.)

(b) Explain why a linear model cannot be appropriate for these times over the long term.

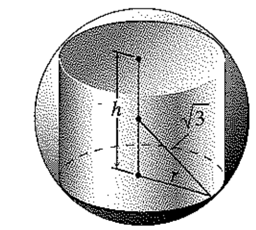
(c) The points appear to be approaching a horizontal asymptote of $y = 52$. What would this mean about the times in this Olympic event?

(d) Subtract 52 from all the times so that they will approach an asymptote of $y = 0$. Redo the scatter plot with the new y-values. Now find the *exponential* regression curve and superimpose its graph on the vertically-shifted scatter plot.

(e) According to the regression curve, what will be the winning time in the women's 100-meter freestyle event at the 2008 Olympics?

67. Inscribing a Cylinder Inside a Sphere A right circular cylinder of radius r and height h is inscribed inside a sphere of radius $\sqrt{3}$ inches.

(a) Use the Pythagorean Theorem to write h as a function of r.



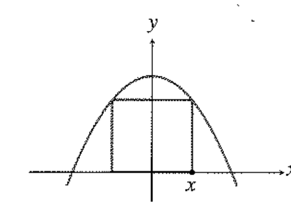
(b) Write the volume V of the cylinder as a function of r.

(c) What values of r are in the domain of V?

(d) Sketch a graph of $V(r)$ over the domain $[0, \sqrt{3}]$.

(e) Use your grapher to find the maximum volume that such a cylinder can have.

68. Inscribing a Rectangle Under a Parabola A rectangle is inscribed between the x-axis and the parabola $y = 36 - x^2$ with one side along the x-axis, as shown in the figure below.



(a) Let x denote the x-coordinate of the point highlighted in the figure. Write the area A of the rectangle as a function of x.

(b) What values of x are in the domain of A?

(c) Sketch a graph of $A(x)$ over the domain.

(d) Use your grapher to find the maximum area that such a rectangle can have.